JANUARY 2005

EXAMINATION
MATHEMATICS
Paper 02 – General Proficiency

2 hours 40 minutes

04 JANUARY 2005 (a.m.)

INSTRUCTIONS TO CANDIDATES

1. Answer ALL questions in Section I, and any TWO from Section II.
2. Write your answers in the booklet provided.
3. All working must be shown clearly.
4. A list of formulae is provided on page 2 of this booklet.

Examination Materials

Electronic calculator (non-programmable)
Geometry set
Mathematical tables (provided)
Graph paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

LIST OF FORMULAE

Volume of a prism

$V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of a right pyramid

$V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference

$C = 2\pi r$ where r is the radius of the circle.

Area of a circle

$A = \pi r^2$ where r is the radius of the circle.

Area of trapezium

$A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations

If $ax^2 + bx + c = 0$,

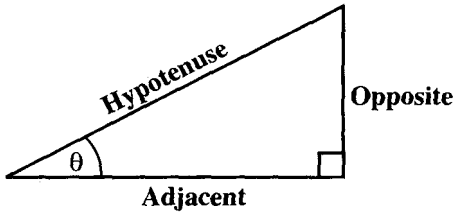
then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

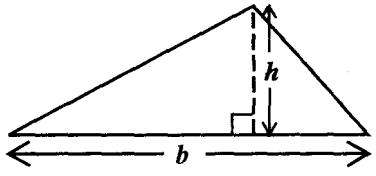
$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



Area of triangle

Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height

Area of $\Delta ABC = \frac{1}{2}ab \sin C$



Area of $\Delta ABC = \sqrt{s(s - a)(s - b)(s - c)}$

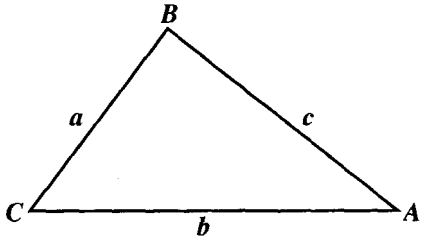
where $s = \dfrac{a + b + c}{2}$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$



SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, evaluate

$$\sqrt{\frac{13.2}{0.33}},$$

writing your answer correct to 3 decimal places. **(3 marks)**

(b) Kim has two telephones. One is cellular and the other is a land line. The rates for local calls are shown in the table below.

Rates	Type of Telephone	
	Cellular	Land Line
Monthly Rental Fee	\$0	\$45
Charge per minute on calls made	85 cents	15 cents

(i) In one month, calls were made lasting for a total of 1 hour and 5 minutes. Show by calculations, that the cost for using the land line telephone was less than the cost for using the cellular telephone.

(ii) For the month of March, the land line telephone was used, and the bill was \$54.60.

Calculate the total time, in minutes, for which the calls lasted. **(8 marks)**

Total 11 marks

2. (a) Given that $r = \frac{2p^2}{q - 3}$,

(i) calculate the value of r when $p = 6$ and $q = 12$.

(ii) rearrange the formula to make q the subject. **(4 marks)**

(b) Factorize completely

(i) $3g - 3t + 2mg - 2mt$

(ii) $3x^2 + 2x - 8$

(iii) $3x^2 - 27$ **(6 marks)**

(c) Given that y varies inversely as x, use the values of x and y from the following table to calculate the value of a.

x	2	32
y	8	a

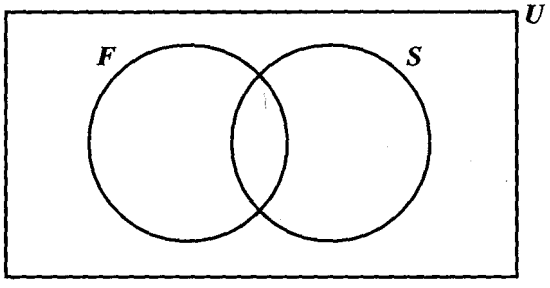
(2 marks)

Total 12 marks

3. (a) 32 candidates took examinations at a CXC examinations centre.

11 took French (F)
9 took Spanish (S)
x took both French and Spanish
18 took neither French nor Spanish

(i) Copy and complete the following Venn diagram to represent the information.

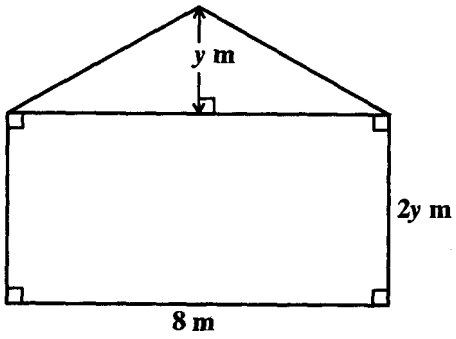


(ii) Write an equation in x for the number of candidates in the universal set.

(iii) Calculate the value of x.

(iv) Shade the region $F' \cap S$. **(6 marks)**

(b) The diagram below, **not drawn to scale**, shows the vertical cross section of a shed.



(i) Write an expression in terms of y for the area of the figure shown.

(ii) Calculate the value of y if the area of the figure is 28 m^2.

(4 marks)

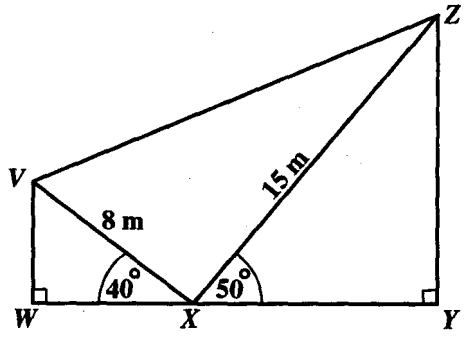
Total 10 marks

4. (a) Using a ruler, a pencil and a pair of compasses only, construct the *rectangle PQRS* in which $PQ = 8$ cm and $PS = 6$ cm.

Measure and state the length of the diagonal, in centimetres. **(6 marks)**

(b) In the diagram below, **not drawn to scale**, WXY is a straight line with Y due east of W, and V due north of W.

Calculate



(i) $\angle ZXV$

(ii) $\angle ZVX$

(iii) the length of VZ

(iv) the bearing of V from X. **(6 marks)**

Total 12 marks

5. (a) The functions f and g are such that

$f(x) = \dfrac{2x + 5}{x - 4}$ and $g(x) = 2x - 3$.

Calculate the value of

(i) $g(4)$

(ii) $fg(2)$

(iii) $g^{-1}(7)$. **(5 marks)**

(b) Write as a single fraction in its simplest form

$\dfrac{3}{x} + \dfrac{4}{x + 1}$. **(3 marks)**

(c) Calculate the value of

$9^{1/2} \times 8^{2/3} \times 4^{0}$ **(3 marks)**

Total 11 marks

6. (a) A straight line is drawn through the points A (1, 1) and B (5, –2).

(i) Calculate the gradient of the line AB.

(ii) Write down the gradient of any line that is perpendicular to AB.

(iii) Determine the equation of the line which passes through D (3,2) and is perpendicular to AB.

Write your answer in the form: $y = mx + c$. **(5 marks)**

(b) An answer sheet is provided for this question.

On the answer sheet provided, draw on the given axes

(i) a triangle with coordinates (2, 1), (3, 3) and (4, 3). Label it A.

(ii) the image of triangle A after a reflection in the line $y = -1$. Label it B.

(iii) the image of triangle A after a translation by the vector $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$. Label it C.
(7 marks)

Total 12 marks

7. The graph shown below is the cumulative frequency curve for the marks scored on a test by a class of 80 students.

Use the graph to estimate

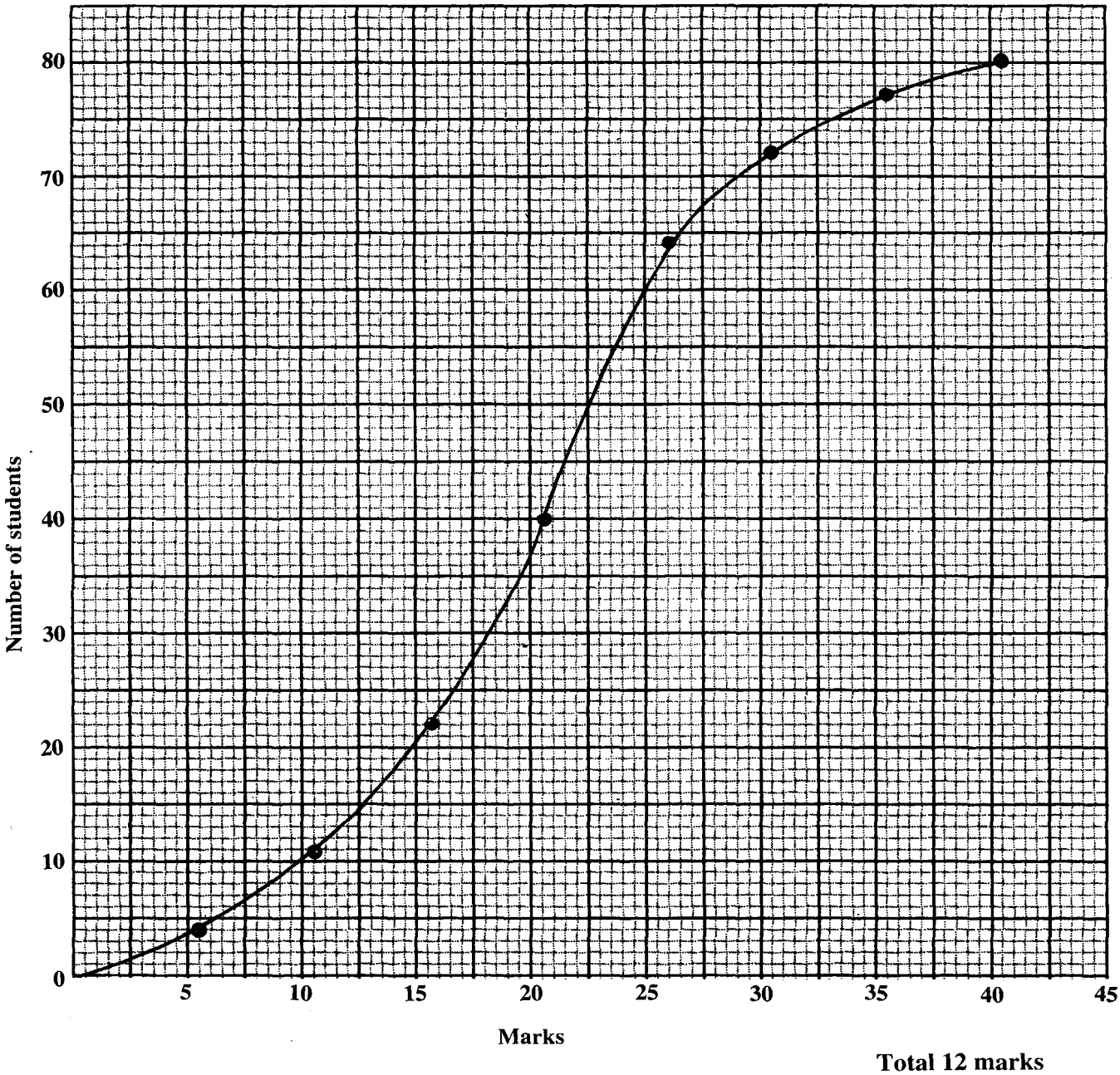
(i) the number of students who scored less than 23 marks **(2 marks)**

(ii) the number of students who scored more than 17 marks **(2 marks)**

(iii) the interquartile range of the marks scored **(3 marks)**

(iv) the probability that a randomly chosen student from the class scored between 17 marks and 23 marks **(3 marks)**

(v) the value of x if only 30 students from the class scored more than x marks. **(2 marks)**



Total 12 marks

8.

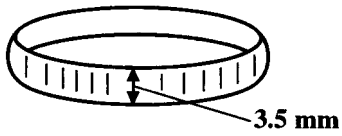


Diagram A

Diagram A, **not drawn to scale**, shows a link from a chain. Each link is a cylindrical ring of thickness 3.5 mm.

Diagram B shows the cross section of the ring. Each ring has internal diameter 14 mm and external diameter 16 mm.

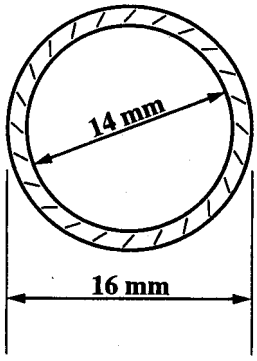


Diagram B

(a) Taking $\pi = 3.14$, calculate the volume of metal in a single link of chain, writing your answer correct to 3 significant figures. **(4 marks)**

(b)

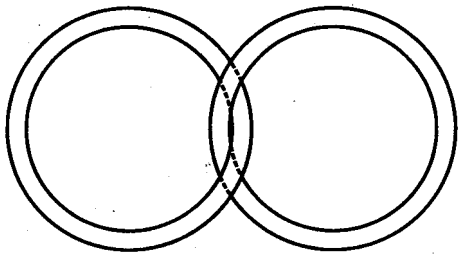


Diagram C

Two of the rings are linked as shown in Diagram C. Show that the length of the chain is 16 mm + 14 mm. **(2 marks)**

(c) Copy and complete the table below which shows the length of the chain formed when rings are linked in a straight line.

Number of rings	Length of chain (in mm)
1	16
2	30
3	44
6	—
—	170

(4 marks)

Total 10 marks

SECTION II

Answer TWO questions in this section.

RELATIONS, FUNCTIONS AND GRAPHS

9. (a) Solve the pair of simultaneous equations

$x^2 = 4 - y$
$x = y + 2$ **(5 marks)**

(b) By simplifying, show that

$(2x - 3)(2x + 3) - (x - 4)^2 \equiv 3x^2 + 8x - 25$ **(2 marks)**

(c) (i) Write $3x^2 + 8x - 25$ in the form $a(x + h)^2 + k$ where a, h and k are real numbers.

(ii) Hence, or otherwise, determine the minimum value of $3x^2 + 8x - 25$. **(5 marks)**

(d) Solve the equation

$3x^2 + 8x - 25 = 0$

giving your answers correct to one decimal place. **(3 marks)**

Total 15 marks

10. Miss James buys x calculators and y folders to sell at a school.

She must buy *at least* 5 calculators.

(i) Write an inequality to represent this information. **(1 mark)**

The number of folders she buys must be *at least* twice the number of calculators.

(ii) Write an inequality to represent this information. **(2 marks)**

A calculator costs \$20 and a folder costs \$5. Miss James must spend *no more* than \$300.

(iii) Write an inequality to represent this information. **(2 marks)**

(iv) a) Using a scale of 2 cm to represent 5 calculators on the x-axis and 2 cm to represent 10 folders on the y-axis, draw the graphs of the lines associated with the inequalities at (i), (ii) and (iii) above.

b) Identify, by shading, the region which satisfies all three inequalities.
(6 marks)

The profit on each calculator is \$6 and on each folder is \$2.

(v) Write an expression in x and y for the total profit, P. **(1 mark)**

Using your graph

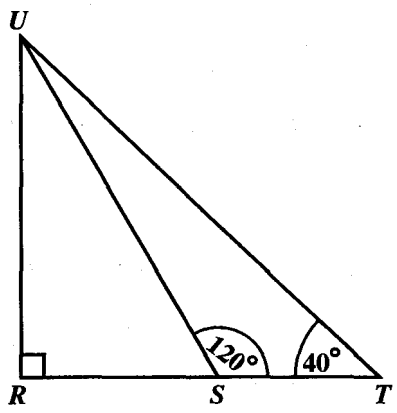
(vi) Write down the coordinates of the vertices of the shaded region. **(1 mark)**

(vii) Calculate the maximum profit. **(2 marks)**

Total 15 marks

GEOMETRY AND TRIGONOMETRY

11. (a) In the diagram below, **not drawn to scale**, *RST* is a horizontal straight line and *UR* represents a vertical pole.



$ST = 15$ m, $\angle UST = 120^\circ$ and $\angle STU = 40^\circ$.

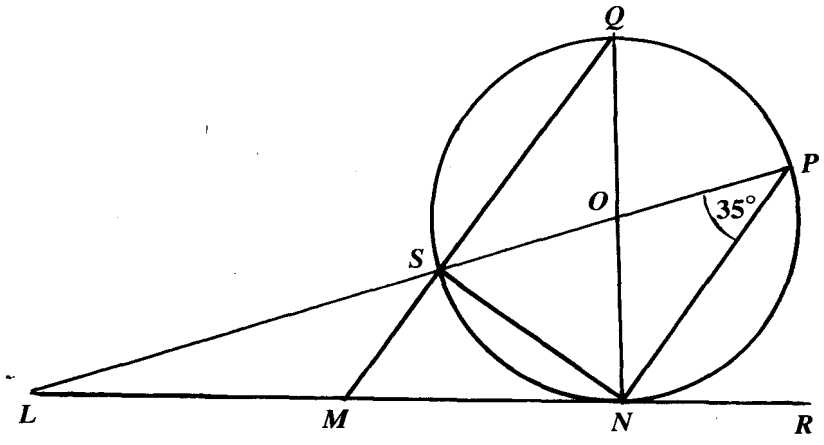
Calculate

(i) the angle of elevation of *U* from *S*.

(ii) the length of *UT*.

(iii) the length of *RU*. **(7 marks)**

(b) The diagram below shows a circle, centre *O*. *LMNR* is a tangent to the circle. *LSOP*, *NOQ* and *MSQ* are straight lines. $\angle SPN = 35^\circ$.



Calculate, giving reasons for each step of your answer,

(i) $\angle SON$

(ii) $\angle NMQ$

(iii) $\angle PLN$

(iv) $\angle SNM$ **(8 marks)**

Total 15 marks

12. (a) For this question, take the radius of the earth to be 6370 km and $\pi = \frac{22}{7}$.

Two points, A and B, are located on the surface of the earth at (45°N, 40°E) and (45°N, 20°W) respectively.

(i) Draw a diagram to represent the earth showing the equator, the line of O° longitude, and points A and B. **(4 marks)**

(ii) Calculate the shortest distance between A and B measured along their common circle of latitude. **(4 marks)**

(b) (i) Given that $y = 2 - \cos x$, copy and complete the table below.

x	0°	30°	60°	90°	120°	150°	180°
y		1.1	1.5		2.5		3

(ii) Using a scale of 2 cm to represent 30° on the x-axis, and 1 cm to represent 0.2 on the y-axis, draw the graph of $y = 2 - \cos x$ for $0° \leq x \leq 180°$.

(iii) Using the graph, or otherwise, determine the value of x for which

$2 - \cos x = 1.8$. **(7 marks)**

Total 15 marks

VECTORS AND MATRICES

13. (a) The position vector of a point P, relative to an origin O, is given as $\overrightarrow{OP} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

$m = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $n = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ are two vectors in the same plane as $\overrightarrow{OP}$.

Given that $\overrightarrow{PQ} = m + 2n$

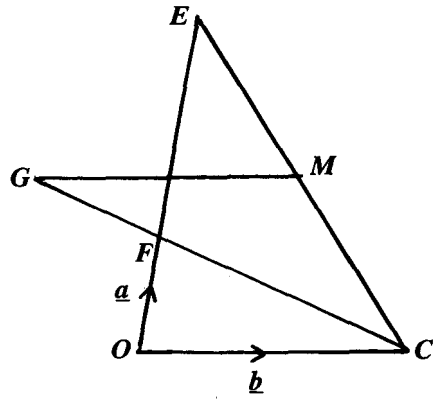
Calculate

(i) $\overrightarrow{PQ}$, writing your answer in the form $\begin{pmatrix} x \\ y \end{pmatrix}$

(ii) $|\overrightarrow{PQ}|$. **(3 marks)**

(b) In the diagram below, **not drawn to scale**, M is the midpoint of CE.

$\overrightarrow{OF} = \underline{a}$, $\overrightarrow{OC} = \underline{b}$ and $\overrightarrow{FE} = 2\overrightarrow{OF}$.



Express in terms of $\underline{a}$ and $\underline{b}$ in simplified form

(i) $\overrightarrow{CF}$ **(2 marks)**

(ii) $\overrightarrow{CE}$ **(2 marks)**

(iii) $\overrightarrow{CM}$. **(2 marks)**

The point G is on CF produced so that $\overrightarrow{CG} = k\overrightarrow{CF}$ where k is a scalar.

(iv) Express $\overrightarrow{MG}$ in terms of $\underline{a}$, $\underline{b}$, and k. **(3 marks)**

(v) Determine the value of k for which $\overrightarrow{MG} = \overrightarrow{CO}$. **(3 marks)**

Total 15 marks

14. (a) (i) Find the inverse of the matrix $M = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$. **(2 marks)**

(ii) Calculate the values of x and y for which $M \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 1 \end{bmatrix}$. **(4 marks)**

(b) Under a transformation T, represented by the matrix $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$, the points A (– 4, 2) and B (– 2, 5) are mapped onto A' (–2, 4) and B' (–5, 2) respectively.

Using a matrix method,

(i) Determine the values of p, q, r and s. **(6 marks)**

(ii) Calculate the coordinates of the point C' which is the image of C (– 2, 2) under T. **(3 marks)**

Total 15 marks

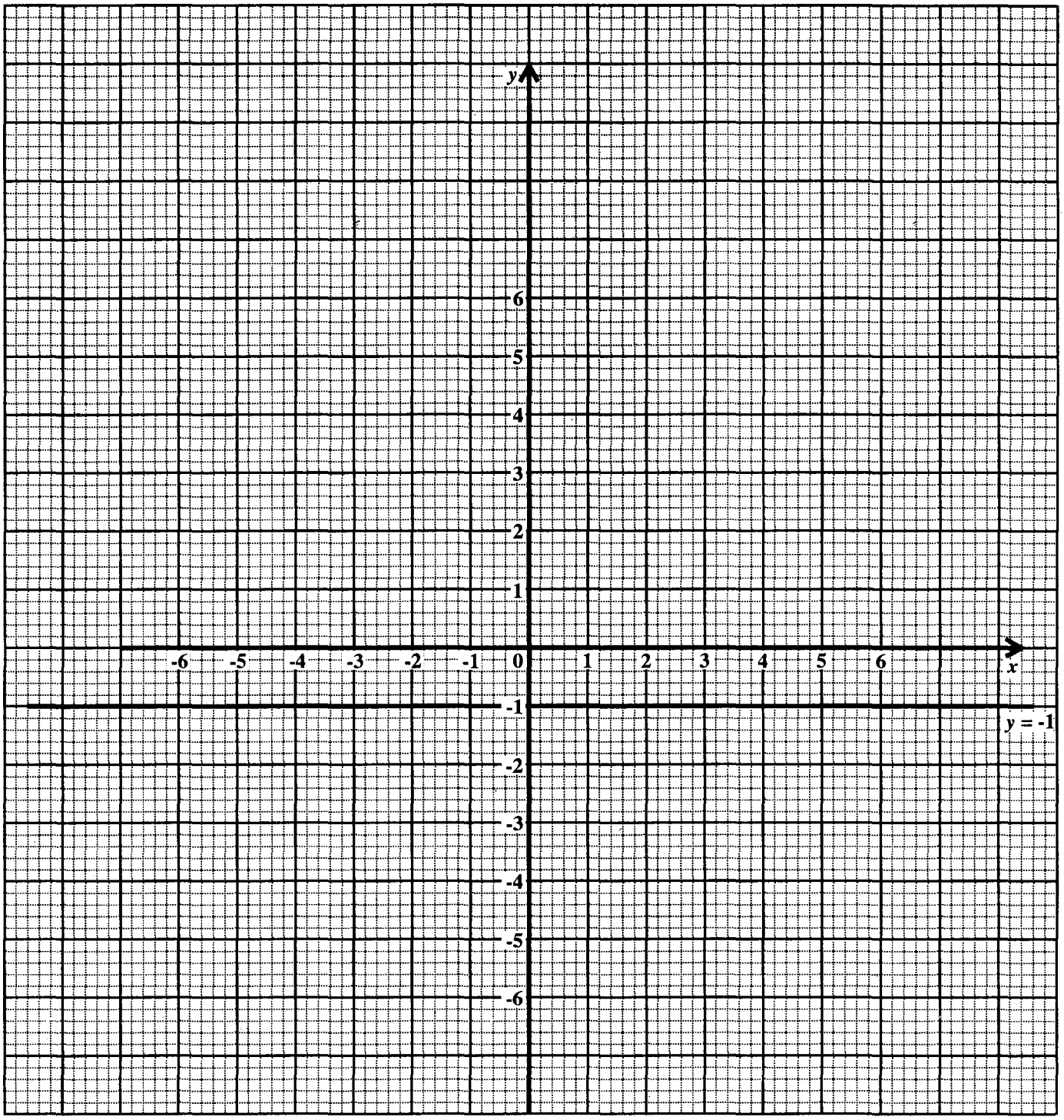
END OF TEST

JANUARY 2005

MATHEMATICS

Paper 02 – General Proficiency

Answer Sheet for Question 6 (b) **Candidate Number .....**



ATTACH THIS ANSWER SHEET TO YOUR BOOKLET